



Problem 1. Jack has his own garden. There are 15 potted flowers arranged in a straight line, spaced 1 meter apart. Additionally, on the same line, 2 meters before the first flower, there is a water well. Jack has a watering can with a capacity of 5 liters, which he fills from the well and then uses to water the flowers in any order he chooses, using 1 liter per flower. To carry 1 liter of water for 1 meter, Jack uses 1 joule of energy. He starts at the well by filling the watering can to full capacity and then waters 5 flowers. When the water runs out, he returns to the well. He repeats the process until all the flowers are watered. Jack is lazy, so he wants to use as little energy as possible to carry the water (carrying the empty can is ignored). What is the minimum number of joules Jack will use? Provide an example route to make his dream come true.

Author of the problem: Michał Fronczek

Solution: Let us assume the existence of *destiny* and assign each liter of water a specific flower that it will eventually water. In this way, instead of considering the movement of the entire watering can, we can focus on each individual liter being carried. This is justified by the fact that the amount of energy used is calculated not based on the can itself, but on the individual liters of water it contains — which we can thus treat as separate “entities”. During its journey, each such liter will cover a distance prescribed by Jack, but it will always begin at the well and end at a particular flower. The minimum distance it must travel is the direct segment between these two points. We can repeat this reasoning for each such liter. Hence, the minimum amount of energy (in theory — since we have yet to check whether such a route is physically realizable) that Jack might use is the sum of distances from the well to each flower. The first flower is 2 meters away from the well, and the farthest one (the 15th) is 16 meters away. All other distances are integers between these values, so the total sum is:

$$2 + 3 + \dots + 16 = 1 + 2 + 3 + \dots + 16 - 1 = \frac{16 \cdot (16 + 1)}{2} - 1 = 135.$$

Therefore, the theoretical minimum is **135 joules**.

It remains to show that there exists a route that allows this result to be achieved. Here is one such route:

“After the i -th filling of the watering can at the well, Jack walks to the $(5i - 4)$ -th flower, and then waters the flowers in order: $(5i - 4)$, $(5i - 3)$, $(5i - 2)$, $(5i - 1)$, and $(5i)$.”

A direct check confirms that this route works.

Thus, the final answer is:

135 J



Problem 2. Find a polynomial with integer coefficients that has $\sqrt{2} + \sqrt[3]{3}$ as a root.

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Solution: In my solution, I will construct the desired polynomial starting from the expression $x = \sqrt{2} + \sqrt[3]{3}$, and then apply addition and exponentiation operations on both sides to transform the original equation into a polynomial with integer coefficients, while preserving the intended root.

First, I will show that adding expressions to both sides and raising both sides of an equation to a power preserves existing roots:

- a) If x satisfies the equation $f(t) = g(t)$, then it also satisfies $f(t) + h(t) = g(t) + h(t)$, since subtracting $h(t)$ from both sides returns us to the original equation. Hence, x is a root of the transformed equation as well.
- b) If x satisfies $f(t) = g(t)$, then it also satisfies $f(t)^n = g(t)^n$, because this is equivalent to $f(t)^n - g(t)^n = 0$. By the identity for the difference of powers, the left-hand side is divisible by $f(t) - g(t)$, which has x as a root, so x is also a root of the transformed equation.

Therefore, if x is a root of the initial equation, it will remain a root throughout the transformation steps. Let us now determine the polynomial:

$$\begin{aligned}
 x &= \sqrt{2} + \sqrt[3]{3} \\
 x - \sqrt{2} &= \sqrt[3]{3} \\
 (x - \sqrt{2})^3 &= \sqrt[3]{3}^3 \\
 x^3 - 3 \cdot x^2 \cdot \sqrt{2} + 3 \cdot x \cdot \sqrt{2}^2 - \sqrt{2}^3 &= 3 \\
 x^3 - 3x^2\sqrt{2} + 6x - 2\sqrt{2} &= 3 \\
 x^3 + 6x - 3 &= \sqrt{2} \cdot (3x^2 + 2) \\
 [x^3 + 6x - 3]^2 &= [\sqrt{2} \cdot (3x^2 + 2)]^2 \\
 x^6 + 36x^2 + 9 + 12x^4 - 6x^3 - 36x &= 18x^4 + 24x^2 + 8 \\
 x^6 - 6x^4 - 6x^3 + 12x^2 - 36x + 1 &= 0
 \end{aligned}$$

We know that x is a root of this equation, so one possible polynomial with integer coefficients having $\sqrt{2} + \sqrt[3]{3}$ as a root is:

$$W(x) = x^6 - 6x^4 - 6x^3 + 12x^2 - 36x + 1.$$