



Problem 1. The Fibonacci sequence is defined by the following formula:

$$F_1 = F_2 = 1$$

$$F_{n+2} = F_{n+1} + F_n$$

(for positive integers n)

Determine whether the number F_{2009} is even.

Source: *Zbiór zadań z kółka matematycznego cz.1* Michał Niedźwiedź, wydawnictwo "Omega", 2022

Choice of problem: Antonina Pajek

Solution: It is known that F_1 and F_2 are odd. Therefore, F_3 is even as the sum of two odd numbers, while F_4 and F_5 are odd as the sum of an even and an odd number. From then on, the situation repeats, i.e. the numbers follow the pattern odd, odd, even, odd, odd, even, ... Thus, even numbers appear in positions divisible by 3. Since 2009 is not divisible by 3, it follows that F_{2009} is odd.

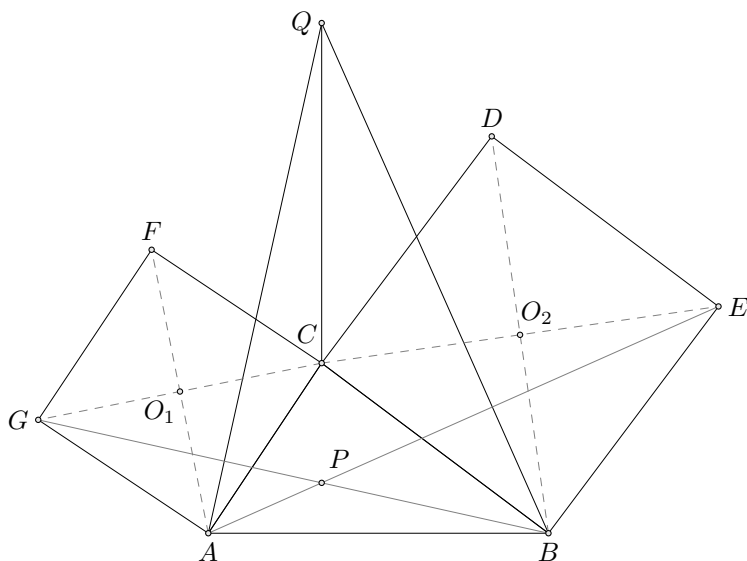


Problem 2. On the sides BC and AC of triangle ABC – constructed outward – we build squares $BCDE$ and $ACFG$. Lines AE and BG intersect at point P . Prove that lines CP and AB are perpendicular.

Source: *Matematyczna liga zadaniowa* Wojciech Klemens, Laura Meissner, Marta Mościcka, Weronika Ormaniec, Tomasz Szymczyk, Wydawnictwa "Omega"

Choice of problem: Antonina Pajek

Solution: Let point O_1 be the center of symmetry of square $ACFG$, and O_2 the center of square $BCDE$. Consider a rotation of the plane around point O_1 by an angle of 90° (counterclockwise).



Under this transformation, point A maps to point C . Let point Q be the image of point B . Then segment CQ has the same length as segment AB and the lines AB and CQ are perpendicular. At the same time, the image of segment GB is segment AQ . Thus $GB \perp AQ$, which means that line GB contains the altitude of triangle ABQ drawn from vertex B .

Similarly, considering the rotation of the plane around point O_2 by an an-

gle of -90° (that is, by 90° clockwise), we obtain that the image of AB is again segment CQ , and that line AE contains the altitude of triangle ABQ drawn from vertex A . Therefore, point P is the intersection point of the altitudes of triangle ABQ . Since lines CQ and PQ are both perpendicular to line AB , they must coincide. Hence line CP is perpendicular to line AB .