Problem 1. In any triangle ABC, let M and N be the midpoints of the sides AB and AC, respectively. Let P be the midpoint of the segment MN. Denote by G the centroid of $\triangle ABC$. Prove that the points A, P, and G are collinear.
Note: *The centroid of a triangle is defined as the intersection point of its medians.*

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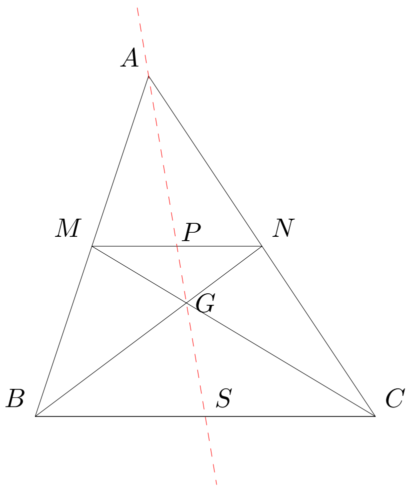
Solution: Let S be the midpoint of segment BC. We then know that AS is a median in $\triangle ABC$, and by definition, point G lies on it. Therefore, A, G, and S are collinear. Hence, we can remove G from our thesis and instead prove that P also lies on the median AS.
Note that $\dfrac{AM}{AB} = \dfrac{AN}{AC} = \dfrac{1}{2}$, and moreover the rays AB and AC share the common point A. Therefore, by the converse of Thales' theorem we obtain that segments MN and BC are parallel. Furthermore, $\dfrac{MN}{BC} = \dfrac{1}{2}$. Now, by the definition of points P and S, we have

$$\frac{MP}{BS} = \frac{\frac{1}{2}MN}{\frac{1}{2}BC} = \frac{MN}{BC} = \frac{1}{2} = \frac{AM}{AB}.$$

Moreover, MP and BS are parallel, since they are segments lying on the parallel lines MN and BC. Therefore, again by the converse of Thales' theorem, we obtain that the points A, P, and S are indeed collinear. As we already know, this completes the proof.



Zadanie 2. Find all prime numbers p such that the number p^4+4^p is also prime.

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Solution: We will consider our number from the point of view of remainders modulo 5.
First, let us check what happens when p is even. The only even prime number is 2, hence $p = 2$. However, in this case the number from the thesis equals 32, which is obviously not prime. Therefore, p must be odd.
Now consider the case when 5 divides p. Since p is prime, this means $p = 5$. Substituting this into the expression from the thesis, we obtain the value 1649. But $1649 = 17 \cdot 97$, so it is not prime. Hence 5 does not divide p.
If p leaves remainder 1 upon division by 5, then $p = 5k + 1$ and

$$p^4 = 625k^4 + 500k^3 + 150k^2 + 20k + 1,$$

so p^4 leaves remainder 1 modulo 5. Similarly, expanding for p that leaves remainders $2, 3, 4$ modulo 5, in every case we find that p^4 always leaves remainder 1.
Now consider powers of 4. They alternate between remainders 4 and 1. Indeed, 4 leaves remainder 4, $4^2 = 16$ leaves remainder 1, $4^3 = 64$ again leaves remainder 4, and so on. In particular, remainder 4 occurs for odd exponents. Since we already know that p is odd, it follows that 4^p leaves remainder 4 modulo 5.
Combining these results, we conclude that the number $p^4 + 4^p$ leaves remainder $1 + 4 = 5$, i.e. 0 modulo 5, so it is divisible by 5. On the other hand, we know that

$$p^4 + 4^p \geqslant 3^4 + 4^3 = 145 > 5,$$

so it cannot be prime, because it has a prime divisor smaller than itself.
Thus we have considered all possible cases, and in none of them did we find a suitable number p. Therefore, such a number cannot exist.