



Problems – 08/25/2025

The solutions to the problems below
will be published on Thursday 08/28/2025

Problem 1. We consider integers modulo n , where $n > 1$. The inverse of an integer $x \pmod{n}$ is an integer y if

$$x \cdot y \equiv 1 \pmod{n}.$$

Prove that the number x has an inverse modulo n if and only if

$$\gcd(n, x) = 1.$$

Problem 2. Marek and Darek each have two identical bottles, partially filled with juice. It is known that if Darek had twice as much juice, he would have more than if Marek received an additional full bottle.

Let m denote the amount of juice Marek has minus the amount contained in one bottle. Similarly, define d for Darek.

Find the smallest possible value of the expression

$$2d^3 + 3d^2m - m^3.$$

Good Luck!

We encourage you to submit your solutions via the website:

[https://mathlovers.eu/submit-solution/!](https://mathlovers.eu/submit-solution/)