



Problem 1. On a right triangle ABC a circumscribed circle with center O and radius R is constructed. On the arc BC (not containing point A) a point D is marked such that triangle ABD is equilateral.

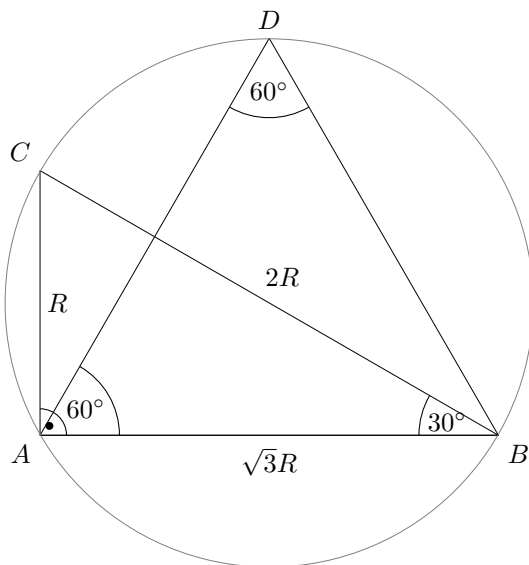
Compute the area of triangle ABC in terms of R .

Author: Maja Chlewicka

Solution: The triangle $\triangle ABD$ is equilateral, therefore each of its angles measures 60° . We can notice that since the angle ADB measures 60° , the angle ACD , based on the same arc AB , also measures 60° . Now we know that since this is a right triangle, its angles measure 60° and 90° , therefore the angle ABC must be 30° . We are dealing with a special triangle, so we know the ratio of its sides. However, we need to calculate its area in terms of the radius of the circle, and since the circle is circumscribed around the right triangle, we have $R = \frac{1}{2}|BC|$ and $|BC| = 2R$. From the properties of a $30^\circ - 60^\circ - 90^\circ$ triangle it follows that $|AC| = R$ and $|AB| = \sqrt{3}R$.

$$P = \frac{1}{2} \cdot \sqrt{3}R \cdot R$$

$$P = R^2 \frac{\sqrt{3}}{2}$$





Problem 2. How many 6-digit numbers are there such that:

- a) No digit is repeated
- b) The digits are arranged in increasing order (from left to right)

Author: Antonina Pajek

Solution: First, consider how many 6-element subsets of the set of digits from 0 to 9 there are (each 6-element subset of digits uniquely determines the number we are looking for) — there are $\binom{10}{6} = 210$ such subsets. Note that if 0 were the smallest of the chosen digits, then the entire number would begin with 0 and thus would not be a 6-digit number. In that case we choose the remaining 5 digits from the other 9 digits; the number of such combinations is $\binom{9}{5} = 126$. Therefore the number of desired 6-digit numbers is

$$210 - 126 = 84.$$