MathLovers

Solutions to the Problems
from 09/04/2025

Problem 1. Find the remainder of the division of the number

$$13^{2024} + 14^{2024} + 15^{2024}$$

by 7.

Source: Antonina Pajek
Solution: Let us consider the remainders of 13, 14, and 15 when divided by 7. We see that:

$$13 \equiv -1 \pmod 7 \qquad 14 \equiv 0 \pmod 7 \qquad 15 \equiv 1 \pmod 7$$

So our expression looks as follows:

$$(-1)^{2024} + 0^{2024} + 1^{2024} \pmod 7.$$

Notice that -1 raised to an even power is equal to 1, so it is enough to sum:

$$1 + 0 + 1 = 2.$$

Thus, the remainder we are looking for is 2.

Problem 2. On farm A there are 12 cows, each of which gives 15 liters of milk per day. On farm B there are 6 fewer cows than on farm A, and each cow from farm B gives 11 liters more milk per day than a cow from farm A. One day the farmers decided to exchange cows:

a) 4 cows were transferred from farm A to farm B

b) 2 cows were transferred from farm B to farm A.

The cows do not change their milk yield, regardless of which farm they are on. Calculate how much milk per day farm A and farm B will now produce after this exchange, and how many cows there are on each farm.

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Solution: Farm A: 12 cows, each giving 15 liters of milk per day, so together they give $12 \cdot 15 = 180$ liters per day. Farm B: 6 cows fewer, i.e. $12 - 6 = 6$, and 11 liters more per cow, i.e. $15 + 11 = 26$ liters, so $26 \cdot 6 = 156$ liters of milk per day.

From farm A to farm B were transferred 4 cows with yield 15 l per day, and from farm B to farm A were transferred 2 cows with yield 26 l per day.

Farm A: $12 - 4 + 2 = 10$ cows. Farm B: $6 - 2 + 4 = 8$ cows.

On farm A there remain 8 cows giving 15 l and 2 cows arriving giving 26 l, so

$$15 \cdot 8 + 2 \cdot 26 = 172 \text{ l per day.}$$

On farm B there are 4 cows giving 26 l and 4 giving 15 l, so

$$4 \cdot 26 + 4 \cdot 15 = 164 \text{ l per day.}$$

Answer: On farm A there are 10 cows producing 172 liters of milk per day, and on farm B 8 cows producing 164 liters per day.