

Problems -09/11/2025

The solutions to the problems below will be published on Sunday 09/14/2025

Problem 1. On the board, the numbers

$$1, 2, 3, \ldots, 2025$$

are written. Scarlett and Antoni play a game. A move consists of replacing three numbers according to the rule

$$(a,b,c) \longrightarrow (a+b-c, b+c-a, c+a-b).$$

The winner is the player after whose move all the numbers on the board are equal. Scarlett moves first. Determine who has a winning strategy.

Problem 2. Prove that for any positive real numbers x, y, z such that $(x+y)(y+z)(z+x) \neq 0$, the following inequality holds:

$$\frac{(1+x^2)(1+y^2)}{2(x+y)} + \frac{(1+y^2)(1+z^2)}{2(y+z)} + \frac{(1+z^2)(1+x^2)}{2(z+x)} \geqslant x+y+z.$$

Good Luck!

We encourage you to submit your solutions via the website: https://mathlovers.eu/submit-solution/!