MathLovers

Solutions to the Problems from 09/18/2025

Problem 1. Kaja, every evening while falling asleep, counts in her head the sheep jumping over the fence. Kaja knows that the number of sheep she counted up to last evening before falling asleep was even. She also knows that she certainly would not have counted up to two hundred sheep and that she certainly counted to more than one hundred. If the number of sheep Kaja counted is a perfect square of a natural number and is not divisible by 3, then to how many sheep did Kaja count last evening?

Author: Scarlett Lafa
Answer: 196
Solution: This number lies in the interval (100, 200). The squares of natural numbers in this interval are: 121, 144, 169, 196. The number must be even, so we are left with 144 and 196. The number must not be divisible by 3, so the only one left is 196.

Problem 2. he probability that it will rain today is 20%.
The probability that it will rain tomorrow is 70%.
The probability that neither today nor tomorrow it will rain is 10%.

(a) Calculate the probability that it will rain both today and tomorrow.

(b) You know that it will not rain today. What is the probability that it will rain tomorrow?

⚠ Note! These events are *not* independent.

Author: Scarlett Lafa
Answers:
a) 0%
b) 100%

Solution:
a) Let us denote the probability of rain today as $P(D)$, no rain today as $P(D')$, and analogously for tomorrow.
Let us construct a table:

	rain tomorrow $P(J)$	no rain tomorrow $P(J')$
rain today $P(D)$	$a\%$	$c\%$
no rain today $P(D')$	$b\%$	10%

This table presents the probability distribution and the dependencies between the events; it is important to notice that all values in it sum to 100%. Knowing that $P(D) = 20\%$, $P(J) = 70\%$ and $P(J' \cap D') = 10\%$, we can construct the system of equations:

$$a + c = 20$$

$$a + b = 70$$

$$b + 10 = 80 \quad (\text{since } P(D') = 80\%)$$

We obtain:

$$b = 70$$

$$a = 0$$

$$c = 20$$

Answer: The probability that it will rain both today and tomorrow is 0%.

b) Since it will not rain today, the entire part of our table devoted to today's rain is irrelevant. We obtain a reduced table:

	rain tomorrow $P(J)$	no rain tomorrow $P(J')$
rain today $P(D)$	0%	20%

But if we know these are our only options, then the probabilities should sum to 100% (just as in the previous table).

Therefore,

$$P(J) = 0\% \quad \text{and} \quad P(J') = 100\% \quad (\text{since } \tfrac{0}{20+0} = 0 \text{ and } \tfrac{20}{20+0} = 1).$$

Answer: 100%