Solutions to the problems
from 30.09.2025

Problem 1. In a team preparing a surprise for Boys' Day, there are 16 girls, each from a different grade and a different study profile. Can they be seated in 4 rows and 4 columns in such a way that in no row or column are there two girls from the same grade or the same profile?

Author: Grzegorz Rudnicki
Solution: Yes, they can. We present an example solution:

$1A$	$2C$	$3D$	$4B$
$2B$	$1D$	$4C$	$3A$
$3C$	$4A$	$1B$	$2D$
$4D$	$3B$	$2A$	$1C$

Problem 2. As part of the surprise, a volleyball tournament was organized, in a round-robin system (each team plays against every other team). Volleyball is played until one team wins three sets. In the tournament, n teams participated and a total of s sets were played. Prove that:

$$5 \cdot \binom{n}{2} \leqslant s \leqslant 3 \cdot \binom{n}{2}$$

The Newton symbol (binomial coefficient) is defined as

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

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Solution: First, we count how many matches were played. The first team in a pair can be chosen in n ways, the second in $n-1$ ways, but since the pair A, B is considered the same as B, A, we must divide our result by 2. Notice that

$$\frac{n \cdot (n-1)}{2} = \frac{1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-2) \cdot (n-1) \cdot n}{1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-2) \cdot 2} = \frac{n!}{(n-2)! \cdot 2!} = \binom{n}{2}$$

If in every match all possible sets were played, then there would be 5 times more sets than matches. If instead the minimum possible number of sets were played, there would be 3 times as many sets as matches. Therefore,

$$5 \cdot \binom{n}{2} \leqslant s \leqslant 3 \cdot \binom{n}{2}.$$

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