Problem 1. During the Chopin Competition, the jury evaluates pianists on a scale from 1 to 25 points. Assume that in the first stage there are n participants.

Each juror gives each participant an integer score from the interval $[1, 25]$. A participant's result is defined as the arithmetic mean of the scores after discarding the highest and the lowest one.

Let $a_1, a_2, \ldots, a_k$ be the scores of the same pianist from k jurors.

Decide whether there exist integers $a_1, \ldots, a_k$ such that, after removing the largest and the smallest value, the average **does not change** compared to the situation when no score is removed.

Author: Tomasz Kossakowski
Solution: Such integers do exist. For example:

$$a_1 = n - 1, \qquad a_2 = a_3 = \ldots = a_{k-1} = n, \qquad a_k = n + 1,$$

where n is any integer in the interval $[2, 24]$. Both before and after removing the extreme scores, the average remains equal to n.

Problem 2. Let X, Y be the projections of points B, C respectively onto the tangent at point A to the circumcircle of triangle ABC. Let M be the midpoint of BC. Prove that $|MX| = |MY|$.

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Solution: Let D be the foot of the perpendicular from M onto the given tangent. Then the lines BX, MD, and CY are parallel, since they are all perpendicular to the tangent. By Thales' theorem, we have

$$\frac{XD}{DY} = \frac{BM}{MC} = 1.$$

Hence $|XD| = |DY|$, and therefore triangles XDM and YDM are congruent (since $\sphericalangle XDM = \sphericalangle YDM = 90^\circ$ and they share the side DM), which completes the proof. ■