MathLovers

Problems for 10/13/2025

Solutions to the following problems
will be published on Thursday 10/16/2025

Problem 1. A swimmer traveled upstream with speed $v = 10\frac{\text{km}}{\text{h}}$. Then he turned back and covered the same distance downstream. Show that he cannot achieve an average speed of $20\frac{\text{km}}{\text{h}}$ for the entire trip (upstream and downstream).

Problem 2. Let (a_i) be a sequence of natural numbers and let p be a prime number such that for $i \in \mathbb{Z}_+$ the recurrence

$$a_{i+1} = a_i^p + 1$$

holds. Prove that, regardless of the choice of the initial term a_1 and the prime p, the sequence will always contain a composite number.

Good luck!

We encourage you to submit your solutions via
https://mathlovers.eu/submit-solution/!