

-  **Problem 1.** Grandma bakes spherical doughnuts of radius R . Inside each doughnut, there is an empty spherical cavity of radius r . The doughnut content in a doughnut (the ratio of the volume occupied by dough to the total volume of the doughnut) is $\frac{1}{2}$. Find r as a function of R .

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Solution: The volume of the empty cavity is $\frac{4}{3}\pi r^3$, and the total volume is $\frac{4}{3}\pi R^3$. Thus, the volume of the outer doughnut part is $\frac{4}{3}\pi(R^3 - r^3)$. Since from the problem statement

$$\frac{\cancel{\frac{4}{3}}\pi r^3}{\cancel{\frac{4}{3}}\pi(R^3 - r^3)} = 1,$$

we know that

$$r^3 = R^3 - r^3.$$

This equation has one real solution; it is

$$r = \frac{\sqrt[3]{2}}{2}R.$$

 **Problem 2.** Grandma plans to serve coffee with milk. In the cupboard there are cups shaped like cones of height H . To what height h should the coffee be poured so that the coffee content in coffee (defined analogously to the doughnut content) equals $\frac{1}{2}$?

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Solution: The coffee must fill half of the cup. Let S be the similarity scale factor between the cone filled with coffee and the whole cone. The ratio of their volumes is S^3 . Since the volume of a cone is directly proportional to its height, we have $Sh = H$, which yields

$$h = \frac{1}{\sqrt[3]{2}}H.$$