



Problem 1. Snow White has chocolates with masses 1 g, 2 g, 3 g, ..., 1001 g and many dwarves. She wants to distribute the chocolates to each of them in such a way that the sum of the masses of the chocolates (excluding the heaviest one) that a dwarf receives must be equal to the mass of the largest chocolate he possesses. Decide whether Snow White is able to distribute the chocolates in such a way.

Author of the problem: Antonina Pajek

Solution: No, Snow White is not able to distribute the chocolates in such a way. Notice that the total mass of all chocolates she has is odd, because $1 + \dots + 1001 = \frac{1001 \cdot 1002}{2} = 501\,501$. Also note that the mass of the chocolates that one dwarf will receive is always even, because if we denote by X the mass of the largest (heaviest) chocolate, then by the conditions of the problem the total mass of the remaining chocolates he receives must also be equal to X (so $X + X = 2X$). Therefore we obtain a contradiction, since the total mass of all chocolates the dwarves would receive is even, while the initial total mass is odd.

 **Problem 2.** Find all pairs of integers x, y satisfying the equation:

$$\frac{y}{2} = \frac{3x - 5}{x - 2}.$$

Author of the problem: Maja Chlewicka

Solution: Multiply both sides by 2 and rearrange:

$$y = \frac{6x - 10}{x - 2}$$

$$y = \frac{6(x - 2) + 2}{x - 2}$$

$$y = 6 + \frac{2}{x - 2}$$

The number 6 is an integer, therefore $\frac{2}{x-2}$ must also be an integer. Hence $x - 2 = 1$, or $x - 2 = -1$, or $x - 2 = 2$, or $x - 2 = -2$. Therefore $x = 3$, or $x = 1$, or $x = 4$, or $x = 0$. Next, for each x we compute y :

$$\begin{array}{llll} x = 3 & x = 1 & x = 4 & x = 0 \\ y = 8 & y = 4 & y = 7 & y = 4 \end{array}$$

Answer: The pairs $(3, 8), (1, 4), (4, 7), (0, 4)$ satisfy the equation.